University of Arizona
Student Satellite Project

# Technical Note

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## 1. Document Overview

This document described the kinematic analysis of the ground tracking control mode. During the ground tracking maneuver, the telescope has to be pointed at a ground station on Earth. The desired attitude, angular velocity and angular acceleration are derived for this maneuver. Algorithms for implementing the equations are given at the end.

## 2. Requirements

## 3. Descriptions/Designs/Discussion

### Nomenclature

| | |
|---|---|
| $\mathbf{x}_{\mathrm{G}}$ | :vector from centroid of Earth to ground station |
| $\mathbf{A}_{\mathrm{a}}^{\mathrm{b}}$ | :transformation matrix from frame $a$ to frame $b$ |
| $\boldsymbol{\omega}_{\mathrm{sid}}$ | :sidereal rotation of the Earth |
| $\lambda_{\mathrm{N}}$ | :geocentric latitude ground station |
| $\lambda_{\mathrm{E}}$ | :geocentric longitude ground station |
| $R_{\mathrm{E}}$ | :distance from Earth centroid to ground station |
| $\iota$ | :inclination of the orbit |
| $R_{\mathrm{S}}$ | :radius of the orbit |
| $\omega$ | :argument of perigee |
| $\Omega$ | :angle of right ascension |
| $\nu$ | :true anomaly |
| $\mathbf{x}_{\mathrm{S}}$ | :vector from Earth centroid to satellite |
| $\mathbf{x}_{\mathrm{G/S}}$ | :displacement from the satellite to the ground station |

$\mathbf{u}_{\mathrm{G/S}}$ :unit vector directed from the satellite to the ground station
$\boldsymbol{\omega}_{\mathrm{d}}$ :desired angular velocity of the satellite
$t_{\mathrm{s}}$ :sampling time

## 3.1 Notation and Coordinate Frames

This section introduces the notation and symbols used in this technote. Furthermore all reference frames used in this paper will be defined.

### 3.1.1 Notation

Vectors will be denoted by lowercase boldface letters. Matrices will be denoted by uppercase boldface letters. Scalars are denoted by italic lowercase letters. For example $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ means that the matrix $\mathbf{A}$ multiplied by vector $\mathbf{x}$ equals a scalar $\lambda$ times the same vector $\mathbf{x}$.

In general, let $\tilde{\mathbf{a}}$ denote the *cross-product matrix* of vector $\mathbf{a}$, the $3 \times 3$ skew-symmetric matrix such that $\tilde{\mathbf{a}}\mathbf{b} = \mathbf{a} \times \mathbf{b}$ for any vector $\mathbf{b}$. Algebraically, $\tilde{\mathbf{a}}$ is given by

$$\tilde{\mathbf{a}} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{1}$$

The coordinate frame of reference is denoted by a superscript. For example, the vector $\mathbf{x}^{\mathrm{ECI}}$ is given in coordinates of the ECI-frame.

### 3.1.2 Definition of Reference Frames

The following reference frames are used in this technote.

#### *3.1.2.1 Spacecraft Frame (SCF)*

The origin of the spacecraft fixed SCF frame is at the center of mass of the satellite. The z-axis points along the bore-sight of the telescope axis. The x-axis is perpendicular to the z-axis and points to the center of the first side panel. The y-axis is chosen such that a right-hand orthonormal reference frame is formed. The reference frame is shown in Fig. 1.

#### *3.1.2.2 Desired Frame (D)*

The origin of the desired frame is at the center of mass of the satellite. The axes of the frame form an orthonormal triplet and represent the desired attitude of the satellite. The spacecraft frame and the desired frame are aligned if the attitude error is zero.

#### *3.1.2.3 Earth-Centered Inertial Frame (ECI)*

The ECI frame is fixed with respect to the stars. Its origin is the center of the earth. The z axis points at the celestial pole. The x axis points toward the mean equinox; that is, the direction from the earth to the sun on the first day of spring. The y axis is chosen such

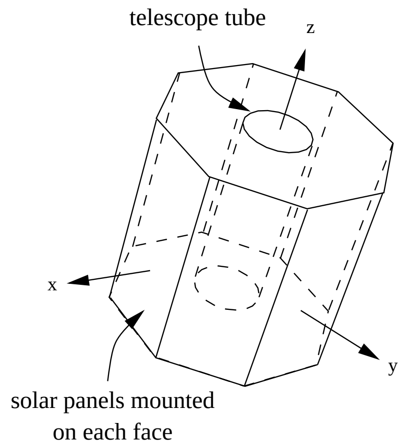


Figure 1: Geometry of the UASat and the spacecraft fixed frame

that a right-hand orthonormal reference frame is formed. This frame is shown in Panel A of Fig. 2.

#### *3.1.2.4 Earth-Fixed Frame (ECF)*

The ECF frame is fixed with respect to the Earth. Its origin is the center of the earth. The z axis points at the celestial pole. The x axis runs through the prime meridian. The y axis is chosen such that a right-hand orthonormal reference frame is formed. This frame is shown in Panel A of Fig. 2.

#### *3.1.2.5 Orbit Frame (ORB)*

Define an orbit coordinate system with its center at the Earth centroid as follows. Let the z-axis correspond to the orbit normal, Let the x-axis be in the direction from the Earth centroid to the ascending node. The y-axis follows from the right hand rule. This frame is shown in Panel B of Fig. 2.

## 3.2 Desired attitude

During the ground tracking maneuver the telescope must be pointing at a ground station on Earth. Therefore the direction from the satellite to the ground station must be known in order to specify a desired attitude. We will first find expression for the location of the ground station and the satellite in ECI coordinates. These expression are subtracted of each other to get the direction.

The location of the ground station in ECI coordinates is

$$\mathbf{x}_{\mathrm{G}}^{\mathrm{eci}} = \mathbf{A}_{\mathrm{ecf}}^{\mathrm{eci}} \mathbf{x}_{\mathrm{G}}^{\mathrm{ecf}} \tag{2}$$

where $\mathbf{A}_{\mathrm{ecf}}^{\mathrm{eci}}$ is the transformation matrix from ECF to ECI coordinates.

Let $\lambda_{\mathrm{E}}$ and $\lambda_{\mathrm{N}}$ be the longitude and geocentric latitude of the ground station respectively. Let $R_{\mathrm{E}}$ be the distance from the Earth centroid to the the ground station. Then the

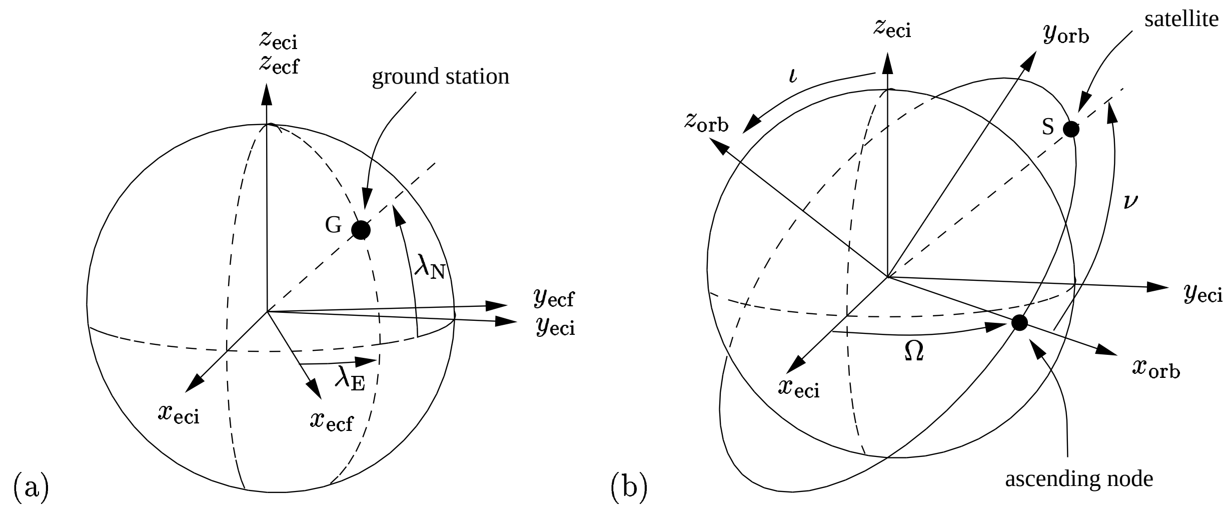


Figure 2: Geometry of ground station on Earth and a circular orbit.

coordinates of the ground station are (see Fig. 2.a)

$$\mathbf{x}_{\mathrm{G}}^{\mathrm{ecf}} = R_{\mathrm{E}} \begin{bmatrix} \cos\lambda_{\mathrm{N}} \cos\lambda_{\mathrm{E}} \\ \cos\lambda_{\mathrm{N}} \sin\lambda_{\mathrm{E}} \\ \sin\lambda_{\mathrm{N}} \end{bmatrix} \tag{3}$$

If we assume that the Earth is round with the geocentric latitude equal to geodetic latitude, the coordinates of the ground station in Tucson are $\lambda_{\mathrm{N}} = 32.19581°$, $\lambda_{\mathrm{E}} = -110.89171°$ and $R_{\mathrm{E}} = 6,378.137$km (equatorial radius). See [1] for an introduction to the geometry of the Earth.

Assume a circular orbit with radius $R_{\mathrm{S}}$ and inclination $\iota$. For a circular orbit is the argument of perigee, $\omega$, not uniquely defined and can be set to zero: $\omega = 0$. Define an orbit coordinate system with its center at the Earth centroid as follows. Let the z-axis correspond to the orbit normal, Let the x-axis be in the direction from the Earth centroid to the ascending node. The y-axis follows from the right hand rule. The orbit and the frame are shown in Panel b of Fig. 2. The satellite orbits in the x-y plane of this frame as

$$\mathbf{x}_{\mathrm{S}}^{\mathrm{orb}} = R_{\mathrm{S}} \begin{bmatrix} \cos\nu \\ \sin\nu \\ 0 \end{bmatrix} \tag{4}$$

where $\nu$ is the true anomaly. The location of the satellite in ECI coordinates is given by

$$\mathbf{x}_{\mathrm{S}}^{\mathrm{eci}} = \mathbf{A}_{\mathrm{orb}}^{\mathrm{eci}} \mathbf{x}_{\mathrm{S}}^{\mathrm{orb}} \tag{5}$$

where the transformation matrix from orbit frame to ECI coordinates is

$$\mathbf{A}_{\mathrm{orb}}^{\mathrm{eci}} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\iota & -\sin\iota \\ 0 & \sin\iota & \cos\iota \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega\cos\iota & \sin\Omega\sin\iota \\ \sin\Omega & \cos\Omega\cos\iota & -\cos\Omega\sin\iota \\ 0 & \sin\iota & \cos\iota \end{bmatrix} \tag{6}$$

where $\Omega$ is the angle of right ascension of the orbit.

Let $\mathbf{x}_{\mathrm{G/S}}$ be the displacement from the satellite to the ground station

$$\mathbf{x}_{\mathrm{G/S}}^{\mathrm{eci}} = \mathbf{x}_{\mathrm{G}}^{\mathrm{eci}} - \mathbf{x}_{\mathrm{S}}^{\mathrm{eci}} \tag{7}$$

The unit vector directed from the satellite to the ground station is

$$\mathbf{u}_{\mathrm{G/S}}^{\mathrm{eci}} = \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\mathbf{x}_{\mathrm{G/S}}^{\mathrm{eci}} \tag{8}$$

where $\|\mathbf{x}_{\mathrm{G/S}}\|$ is the Euclidean norm of vector $\mathbf{x}_{\mathrm{G/S}}$.

The desired attitude should be such that the telescope points toward the ground station. Therefore, the third column of $\mathbf{A}_{\mathrm{d}}$ is determined by $\mathbf{u}_{\mathrm{G/S}}$. The first two column are arbitrary as rotations about the telescope axis are not important; $\mathbf{A}_{\mathrm{d}}$ is not completely specified by the problem.

The desired attitude can be determined as follows. Given initial time $t_0$, let $\mathbf{A}_{\mathrm{d}}(t_0)$ be any orthonormal matrix such that the third column is $\mathbf{u}_{\mathrm{G/S}}(t_0)$. The desired $\mathbf{A}_{\mathrm{d}}(t)$ is determined by integrating the angular velocity:

$$\mathbf{A}_{\mathrm{d}}^{\mathrm{eci}}(t) = \int_{t_0}^{t} \tilde{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{eci}}\mathbf{A}_{\mathrm{d}}^{\mathrm{eci}}(\tau)d\tau + \mathbf{A}_{\mathrm{d}}^{\mathrm{eci}}(t_0) \tag{9}$$

This is only necessary because $\mathbf{A}_{\mathrm{d}}(t)$ is not completely specified by the problem. An suitable desired angular velocity $\boldsymbol{\omega}_{\mathrm{d}}^{\mathrm{eci}}$ will be derived in Sec. 3.3.

In practice, the integration has to be performed only for the first column of $\mathbf{A}_{\mathrm{d}}$. The third column follows from the unit vector $\mathbf{u}_{\mathrm{G/S}}$ and the second column is determined by the cross product of the third and first column $\mathbf{e}_{2,\mathrm{d}} = \tilde{\mathbf{e}}_{3,\mathrm{d}}\mathbf{e}_{1,\mathrm{d}}$. Writing (9) only for the first column

$$\mathbf{e}_{1,\mathrm{d}}^{\mathrm{eci}}(t) = \int_{t_0}^{t} \tilde{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{eci}}\mathbf{e}_{1,\mathrm{d}}^{\mathrm{eci}}(\tau)d\tau + \mathbf{e}_{1,\mathrm{d}}^{\mathrm{eci}}(t_0) \tag{10}$$

In practice this integration will be performed numerically. An update function will be called each $t_{\mathrm{s}}$ seconds to propagate the unit vector using forward Euler integration. The Euler integration rule is

$$\mathbf{e}_{1,\mathrm{d}}^{\mathrm{eci}}((k+1)t_{\mathrm{s}}) = \left[\mathbf{I}_3 + \tilde{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{eci}}(kt_{\mathrm{s}})t_{\mathrm{s}}\right]\mathbf{e}_{1,\mathrm{d}}^{\mathrm{eci}}(kt_{\mathrm{s}}) \tag{11}$$

## 3.3 Desired angular velocity

The desired angular velocity is not uniquely specified for this problem. A particular choice with no rotation component about $\mathbf{u}_{\mathrm{G/S}}$ is

$$\boldsymbol{\omega}_{\mathrm{d}}^{\mathrm{eci}} = \tilde{\mathbf{u}}_{\mathrm{G/S}}^{\mathrm{eci}}\dot{\mathbf{u}}_{\mathrm{G/S}}^{\mathrm{eci}} \tag{12}$$

To compute this angular velocity one needs to compute the time derivative of the unit vector $\mathbf{u}_{\mathrm{G/S}}$. The rate of change of the unit vector follows by differentiating (8) with respect to time

$$\dot{\mathbf{u}}_{\mathrm{G/S}}^{\mathrm{eci}} = \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\dot{\mathbf{x}}_{\mathrm{G/S}}^{\mathrm{eci}} - \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|^2}\frac{d}{dt}(\|\mathbf{x}_{\mathrm{G/S}}\|)\mathbf{x}_{\mathrm{G/S}}^{\mathrm{eci}} \tag{13}$$

The derivative of the Euclidean norm $\|\mathbf{x}_{\mathrm{G/S}}\|$ can be computed by differentiating the square of the norm and then dividing both side by $2\|\mathbf{x}_{\mathrm{G/S}}\|$

$$2\|\mathbf{x}_{\mathrm{G/S}}\|\frac{d}{dt}(\|\mathbf{x}_{\mathrm{G/S}}\|) = \frac{d}{dt}(\|\mathbf{x}_{\mathrm{G/S}}\|^2) = \frac{d}{dt}\big((\mathbf{x}^{\mathrm{eci}}_{\mathrm{G/S}})^t\mathbf{x}^{\mathrm{eci}}_{\mathrm{G/S}}\big) = 2(\mathbf{x}^{\mathrm{eci}}_{\mathrm{G/S}})^t\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} \Rightarrow \tag{14}$$

$$\frac{d}{dt}(\|\mathbf{x}_{\mathrm{G/S}}\|) = \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}(\mathbf{x}^{\mathrm{eci}}_{\mathrm{G/S}})^t\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} \tag{15}$$

Substitution of this result into (13) and using (8) yields

$$\dot{\mathbf{u}}^{\mathrm{eci}}_{\mathrm{G/S}} = \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\big(\mathbf{I}_3 - \mathbf{u}^{\mathrm{eci}}_{\mathrm{G/S}}(\mathbf{u}^{\mathrm{eci}}_{\mathrm{G/S}})^t\big)\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} \tag{16}$$

Note that $\dot{\mathbf{u}}_{\mathrm{G/S}}$ is perpendicular to $\mathbf{u}_{\mathrm{G/S}}$. The velocity vector $\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}}$ can be found by differentiating (2) and (5). In particular, for a fixed ground station it follows from (2) that

$$\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G}} = \dot{\mathbf{A}}^{\mathrm{eci}}_{\mathrm{ecf}}\mathbf{x}^{\mathrm{ecf}}_{\mathrm{G}} = \tilde{\boldsymbol{\omega}}^{\mathrm{eci}}_{\mathrm{sid}}\mathbf{A}^{\mathrm{eci}}_{\mathrm{ecf}}\mathbf{x}^{\mathrm{ecf}}_{\mathrm{G}} = \tilde{\boldsymbol{\omega}}^{\mathrm{eci}}_{\mathrm{sid}}\mathbf{x}^{\mathrm{eci}}_{\mathrm{G}} \tag{17}$$

where $\boldsymbol{\omega}^{\mathrm{eci}}_{\mathrm{sid}} = \begin{bmatrix}0 & 0 & 2\pi/86,164\end{bmatrix}^t$ is the sidereal rotation of the Earth. Similarly, assuming that $\Omega$ and $\iota$ vary only very slowly in time such that the matrix $\mathbf{A}^{\mathrm{eci}}_{\mathrm{orb}}$ can be considered constant, it follows from (5) that

$$\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{S}} = \mathbf{A}^{\mathrm{eci}}_{\mathrm{orb}}\dot{\mathbf{x}}^{\mathrm{orb}}_{\mathrm{S}} = \mathbf{A}^{\mathrm{eci}}_{\mathrm{orb}}R_{\mathrm{S}}\dot{\nu}\begin{bmatrix}-\sin\nu \\ \cos\nu \\ 0\end{bmatrix} \tag{18}$$

where $\dot{\nu}$ is the orbit rate in rad/s. The velocity vector $\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}}$ is the difference of (17) and (18)

$$\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} = \tilde{\boldsymbol{\omega}}^{\mathrm{eci}}_{\mathrm{sid}}\mathbf{x}^{\mathrm{eci}}_{\mathrm{G}} - \mathbf{A}^{\mathrm{eci}}_{\mathrm{orb}}R_{\mathrm{S}}\dot{\nu}\begin{bmatrix}-\sin\nu \\ \cos\nu \\ 0\end{bmatrix} \tag{19}$$

From (12), (16), (19) and using that any vector $\mathbf{a}$ yields $\tilde{\mathbf{a}}\mathbf{a} = \mathbf{0}$, it follows that

$$\boldsymbol{\omega}^{\mathrm{eci}}_{\mathrm{d}} = \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\tilde{\mathbf{u}}^{\mathrm{eci}}_{\mathrm{G/S}}\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} \tag{20}$$

### 3.4 Desired Angular Acceleration

The desired angular acceleration follows by differentiating (20) with respect to time

$$\dot{\boldsymbol{\omega}}^{\mathrm{eci}}_{\mathrm{d}} = \frac{-1}{\|\mathbf{x}_{\mathrm{G/S}}\|^2}\frac{d}{dt}(\|\mathbf{x}_{\mathrm{G/S}}\|)\tilde{\mathbf{u}}^{\mathrm{eci}}_{\mathrm{G/S}}\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} + \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\dot{\tilde{\mathbf{u}}}^{\mathrm{eci}}_{\mathrm{G/S}}\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} + \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\tilde{\mathbf{u}}^{\mathrm{eci}}_{\mathrm{G/S}}\ddot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} \tag{21}$$

Using $\tilde{\mathbf{a}}\mathbf{b} = -\tilde{\mathbf{b}}\mathbf{a}$ and substitution of (15), (16) yields

$$\dot{\boldsymbol{\omega}}^{\mathrm{eci}}_{\mathrm{d}} = \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|^3}(\mathbf{x}^{\mathrm{eci}}_{\mathrm{G/S}})^t\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}}\dot{\tilde{\mathbf{x}}}^{\mathrm{eci}}_{\mathrm{G/S}}\mathbf{u}^{\mathrm{eci}}_{\mathrm{G/S}} - \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|^2}\dot{\tilde{\mathbf{x}}}^{\mathrm{eci}}_{\mathrm{G/S}}\big(\mathbf{I}_3 - \mathbf{u}^{\mathrm{eci}}_{\mathrm{G/S}}(\mathbf{u}^{\mathrm{eci}}_{\mathrm{G/S}})^t\big)\dot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} + \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\tilde{\mathbf{u}}^{\mathrm{eci}}_{\mathrm{G/S}}\ddot{\mathbf{x}}^{\mathrm{eci}}_{\mathrm{G/S}} \tag{22}$$

Using $\tilde{\mathbf{a}}\mathbf{a} = 0$ and (8) one obtains

$$\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{eci}} = \frac{2}{\|\mathbf{x}_{\mathrm{G/S}}\|^2}\dot{\tilde{\mathbf{x}}}_{\mathrm{G/S}}^{\mathrm{eci}}\mathbf{u}_{\mathrm{G/S}}^{\mathrm{eci}}(\mathbf{u}_{\mathrm{G/S}}^{\mathrm{eci}})^t\dot{\mathbf{x}}_{\mathrm{G/S}}^{\mathrm{eci}} + \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\tilde{\mathbf{u}}_{\mathrm{G/S}}^{\mathrm{eci}}\ddot{\mathbf{x}}_{\mathrm{G/S}}^{\mathrm{eci}} \tag{23}$$

Assuming that $\tilde{\boldsymbol{\omega}}_{\mathrm{sid}}^{\mathrm{eci}}$, $\mathbf{x}_{\mathrm{G}}^{\mathrm{eci}}$, $\mathbf{A}_{\mathrm{orb}}^{\mathrm{eci}}$, $R_{\mathrm{S}}$, and $\dot{\nu}$ are constant or slowly time varying, the acceleration follows from differentiation of (19) with respect to time

$$\ddot{\mathbf{x}}_{\mathrm{G/S}}^{\mathrm{eci}} = \mathbf{A}_{\mathrm{orb}}^{\mathrm{eci}} R_{\mathrm{S}}\dot{\nu}^2 \begin{bmatrix} \cos\nu \\ \sin\nu \\ 0 \end{bmatrix} \tag{24}$$

Using this result and (20) it follows from (23) that

$$\dot{\boldsymbol{\omega}}_{\mathrm{d}}^{\mathrm{eci}} = \frac{-2}{\|\mathbf{x}_{\mathrm{G/S}}\|}\boldsymbol{\omega}_{\mathrm{d}}^{\mathrm{eci}}(\mathbf{u}_{\mathrm{G/S}}^{\mathrm{eci}})^t\dot{\mathbf{x}}_{\mathrm{G/S}}^{\mathrm{eci}} + \frac{1}{\|\mathbf{x}_{\mathrm{G/S}}\|}\tilde{\mathbf{u}}_{\mathrm{G/S}}^{\mathrm{eci}}\mathbf{A}_{\mathrm{orb}}^{\mathrm{eci}} R_{\mathrm{S}}\dot{\nu}^2 \begin{bmatrix} \cos\nu \\ \sin\nu \\ 0 \end{bmatrix} \tag{25}$$

## 3.5 Algorithms

The desired attitude, angular velocity and angular acceleration can be computed using the following algorithms. The algorithm `initialize` should be called only once when the ground tracking mode is entered. After initialization, the algorithm `update` should be called each $t_{\mathrm{s}}$ seconds.

**Algorithm 1** ***initialize***

***Input:*** $\mathbf{A}_{sc}^{eci}$, $\mathbf{A}_{ecf}^{eci}$, $\mathbf{x}_{G}^{ecf}$, $\mathbf{A}_{orb}^{eci}$, $\nu$, $\dot{\nu}$, $R_{\mathrm{S}}$, $\omega_{\mathrm{sid}}$,
***Output:*** $\mathbf{A}_{d}^{eci}$, $\boldsymbol{\omega}_{d}^{eci}$

1. *Compute the unit vector* $\mathbf{u}_{\mathrm{G/S}}^{eci}$ *in the direction from the satellite to the ground station.*
   (a) *compute* $\mathbf{x}_{\mathrm{S}}^{orb}$ *using* (4)
   (b) *compute* $\mathbf{x}_{\mathrm{S}}^{eci}$ *using* (5)
   (c) *compute* $\mathbf{x}_{G}^{eci}$ *using* (2)
   (d) *compute* $\mathbf{x}_{\mathrm{G/S}}^{eci}$ *using* (7)
   (e) *compute* $\|\mathbf{x}_{\mathrm{G/S}}\| = \sqrt{(\mathbf{x}_{\mathrm{G/S}}^{eci})^t\mathbf{x}_{\mathrm{G/S}}^{eci}}$
   (f) *compute* $\mathbf{u}_{\mathrm{G/S}}^{eci}$ *using* (8)
2. *Compute the desired orientation* $\mathbf{A}_{d}^{eci}$
   (a) *Set the third column equal to the unit vector towards the ground station* $\mathbf{e}_{3,d}^{eci}(k+1) = \mathbf{u}_{\mathrm{G/S}}^{eci}$
   (b) *If* $\tilde{\mathbf{e}}_{3,d}^{eci}\mathbf{e}_{1,sc}^{eci} \neq 0$ *then set* $\mathbf{e}_{1,d}^{eci} = \mathbf{e}_{1,sc}^{eci}$ *else set* $\mathbf{e}_{1,d}^{eci} = \mathbf{e}_{2,sc}^{eci}$.

(c) *Make this vector orthogonal to the third column by subtracting the component in the direction of the third column vector* $\mathbf{e}^{eci}_{1,d} = \mathbf{e}^{eci}_{1,d} - (\mathbf{e}^{eci}_{3,d})^t \mathbf{e}^{eci}_{1,d} \mathbf{e}^{eci}_{3,d}$. *Normalize the result to obtain a unit vector again.*

(d) *The second column follows from taking the cross product between the third and first columns* $\mathbf{e}^{eci}_{2,d} = \tilde{\mathbf{e}}^{eci}_{3,d} \mathbf{e}^{eci}_{1,d}$

3. *Compute the desired angular velocity* $\boldsymbol{\omega}^{eci}_{d}$

   (a) *compute* $\dot{\mathbf{x}}^{eci}_{\mathrm{G/S}}$ *using* (19)

   (b) *compute* $\boldsymbol{\omega}^{eci}_{d}$ *using* (20)

**Algorithm 2** ***update***

***Input:*** $\mathbf{A}^{eci}_{ecf}$, $\mathbf{x}^{ecf}_{G}$, $\mathbf{A}^{eci}_{orb}$, $\nu$, $\dot{\nu}$, $R_{\mathrm{S}}$, $\omega_{\mathrm{sid}}$,
***Output:*** $\mathbf{A}^{eci}_{d}$, $\boldsymbol{\omega}^{eci}_{d}$, $\dot{\boldsymbol{\omega}}^{eci}_{d}$

1. *Compute the unit vector* $\mathbf{u}^{eci}_{\mathrm{G/S}}$ *in the direction from the satellite to the ground station.*

   (a) *compute* $\mathbf{x}^{orb}_{\mathrm{S}}$ *using* (4)

   (b) *compute* $\mathbf{x}^{eci}_{\mathrm{S}}$ *using* (5)

   (c) *compute* $\mathbf{x}^{eci}_{G}$ *using* (2)

   (d) *compute* $\mathbf{x}^{eci}_{\mathrm{G/S}}$ *using* (7)

   (e) *compute* $\|\mathbf{x}_{\mathrm{G/S}}\| = \sqrt{(\mathbf{x}^{eci}_{\mathrm{G/S}})^t \mathbf{x}^{eci}_{\mathrm{G/S}}}$

   (f) *compute* $\mathbf{u}^{eci}_{\mathrm{G/S}}$ *using* (8)

2. *Compute the desired orientation* $\mathbf{A}^{eci}_{d}$

   (a) *Set the third column equal to the unit vector towards the ground station* $\mathbf{e}^{eci}_{3,d}(k+1) = \mathbf{u}^{eci}_{\mathrm{G/S}}$

   (b) *Compute the new value of the first column* $\mathbf{e}^{eci}_{1,d}(k+1)$ *using* (11) *with the value of the desired angular velocity* $\boldsymbol{\omega}^{eci}_{d}(k)$ *computed in the previous iteration.*

   (c) *Make this vector orthogonal to the third column by subtracting the component in the direction of the third column vector* $\mathbf{e}^{eci}_{1,d} = \mathbf{e}^{eci}_{1,d} - (\mathbf{e}^{eci}_{3,d})^t \mathbf{e}^{eci}_{1,d} \mathbf{e}^{eci}_{3,d}$. *Normalize the result to obtain a unit vector again.*

   (d) *The second column follows from taking the cross product between the third and first columns* $\mathbf{e}^{eci}_{2,d} = \tilde{\mathbf{e}}^{eci}_{3,d} \mathbf{e}^{eci}_{1,d}$

3. *Compute the desired angular velocity* $\boldsymbol{\omega}^{eci}_{d}$

   (a) *compute* $\dot{\mathbf{x}}^{eci}_{\mathrm{G/S}}$ *using* (19)

   (b) *compute* $\boldsymbol{\omega}^{eci}_{d}$ *using* (20)

4. *compute the desired angular acceleration* $\dot{\boldsymbol{\omega}}^{eci}_{d}$ *using* (25)

# 4. Lists

# 5. Interface Requirements and Specifications

# 6. Current Status

# 7. Test Plan

# 8. Concerns and Open Issues

- A perfect spherical Earth is assumed to obtain the ECF coordinates $\mathbf{x}_{\mathrm{G}}^{\mathrm{ecf}}$ of the ground station in Tucson. The coordinates should be estimated more accurately.
- Dr. Wing of the LCS team mentioned that we should take the finite speed of light into consideration and therefore point a little ahead. Initial calcultions indicated that this error will be very small compared the pointing error. The LCS team should analysis this issue and document it.

# 9. References

[1] M. Kayton and W.R. Fried. *Avionics Navigation Systems.* Wiley-Interscience, second edition, 1997.

# Contents