University of Arizona
Student Satellite Project

Technical Note

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1. Document Overview

This document describes the dynamic equations of motion of the UASat.

2. Requirements

3. Descriptions/Designs/Discussion

3.1 Nomenclature

$\mathbf{h}$	:Angular momentum of the satellite
$\mathbf{I}_{\mathrm{sc}}$	:Inertia of the satellite with the reaction wheels unlocked
$\boldsymbol{\omega}_{\mathrm{sc}}$	:Angular velocity of the satellite
$\mathbf{h}_{\mathrm{i,rw}}$	:Angular momentum of the i^{th} reaction wheel
$\mathbf{h}_{\mathrm{rw}}$	:Angular momentum of the reaction wheels
$\omega_{\mathrm{rw,i}}$	:Angular velocity of the i^{th} reaction wheel with respect to the satellite structure.
$\boldsymbol{\omega}_{\mathrm{rw}}$	:Column vector of the angular velocities of the reaction wheels.
$I_{\mathrm{rw,i}}$	:Inertia of the i^{th} reaction wheel
$\mathbf{I}_{\mathrm{rw}}$	:Diagonal matrix with the inertias of the reaction wheels about their spin axis on its diagonal.
$(\cdot)_{\mathrm{abs}}$	:Absolute velocity. Velocity with respect to the stars.
$\mathbf{e}_{\mathrm{i,rw}}$	:Unit vector along the spin axis of the i^{th} reaction wheel.
$\mathbf{E}_{\mathrm{rw}}$	:Reaction wheel configuration matrix.
$\tau_{\mathrm{rw,i}}$	:Torque i^{th} reaction wheel.
$\boldsymbol{\tau}_{\mathrm{rw}}$	:Column vector of reaction wheel torques

$\boldsymbol{\tau}_{\text{ext}}$:External torques acting on the satellite.

3.2 Notation and Coordinate Frames

This section introduces the notation and symbols used in this technote. Furthermore all reference frames used in this paper will be defined.

3.2.1 Notation

Vectors will be denoted by lowercase boldface letters. Matrices will be denoted by uppercase boldface letters. Scalars are denoted by italic lowercase letters. For example $\mathbf{Ax} = \lambda\mathbf{x}$ means that the matrix $\mathbf{A}$ multiplied by vector $\mathbf{x}$ equals a scalar λ times the same vector $\mathbf{x}$.

In general, let $\tilde{\mathbf{a}}$ denote the *cross-product matrix* of vector $\mathbf{a}$, the 3×3 skew-symmetric matrix such that $\tilde{\mathbf{a}}\mathbf{b} = \mathbf{a} \times \mathbf{b}$ for any vector $\mathbf{b}$. Algebraically, $\tilde{\mathbf{a}}$ is given by

$$\tilde{\mathbf{a}} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix} \tag{1}$$

The coordinate frame of reference is denoted by a superscript. For example, the vector $\mathbf{x}^{\text{ECI}}$ is given in coordinates of the ECI-frame.

3.2.2 Definition of Reference Frames

The following reference frames are used in this technote.

3.2.2.1 Spacecraft Frame (SCF)

The origin of the spacecraft fixed SCF frame is at the center of mass of the satellite. The z-axis points along the bore-sight of the telescope axis. The x-axis is perpendicular to the z-axis and points to the center of the first side panel. The y-axis is chosen such that a right-hand orthonormal reference frame is formed. The reference frame is shown in Fig. 1.

3.3 Dynamic model

3.3.1 Angular momentum

The angular momentum of the satellite, $\mathbf{h}$, in spacecraft coordinates is the sum of the angular momentum of the core and the angular momentum of the reaction wheels

$$\mathbf{h}^{\text{scf}} = \mathbf{I}_{\text{sc}}\boldsymbol{\omega}_{\text{sc}}^{\text{scf}} + \mathbf{h}_{\text{rw}}^{\text{scf}} \tag{2}$$

where $\mathbf{h}_{\text{rw}}^{\text{scf}}$ is the angular momentum associated with the reaction wheels, $\mathbf{I}_{\text{sc}}$ is the inertia of the satellite with the reaction wheels unlocked and $\boldsymbol{\omega}_{\text{sc}}^{\text{scf}}$ is the angular velocity of the satellite.

Let $I_{\text{rw,i}}$ be the inertia of the i^{th} reaction wheels and let $(\omega_{\text{rw,i}})_{\text{abs}}$ be its (absolute) angular velocity around its spin axis with respect to the stars. The angular momentum of the i^{th}

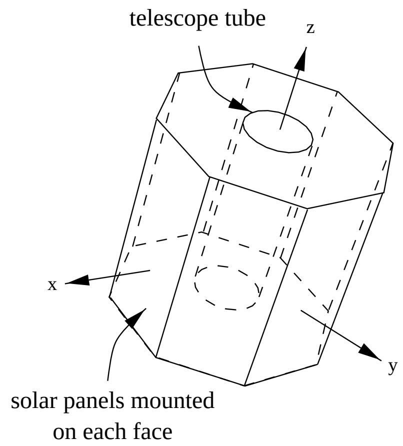

Figure 1: Geometry of the UASat and the spacecraft fixed frame

reaction wheel is $\mathbf{h}^{\text{scf}}_{\text{i,rw}} = I_{\text{rw,i}}(\omega_{\text{rw,i}})_{\text{abs}}\mathbf{e}^{\text{scf}}_{\text{i,rw}}$ where $\mathbf{e}^{\text{scf}}_{\text{i,rw}}$ is the unit vector along the spin axis of the wheel. The absolute angular velocity of the reaction wheel is the sum of its velocity with respect to the satellite structure and the angular velocity of the satellite structure around the spin axis of the reaction wheel $(\omega_{\text{rw,i}})_{\text{abs}} = \omega_{\text{rw,i}} + (\mathbf{e}^{\text{scf}}_{\text{i,rw}})^t\boldsymbol{\omega}^{\text{scf}}_{\text{sc}}$. Hence the angular momentum of the i$^{\text{th}}$ reaction wheel is

$$\mathbf{h}^{\text{scf}}_{\text{i,rw}} = \mathbf{e}^{\text{scf}}_{\text{i,rw}} I_{\text{rw,i}}(\omega_{\text{rw,i}} + (\mathbf{e}^{\text{scf}}_{\text{i,rw}})^t\boldsymbol{\omega}^{\text{scf}}_{\text{sc}}) \tag{3}$$

The total angular momentum of the reaction wheels is the sum of the individual angular momentums and can be expressed in matrix form as

$$\mathbf{h}^{\text{scf}}_{\text{rw}} = \mathbf{E}_{\text{rw}}\mathbf{I}_{\text{rw}}(\boldsymbol{\omega}_{\text{rw}} + (\mathbf{E}_{\text{rw}})^t\boldsymbol{\omega}^{\text{scf}}_{\text{sc}}) \tag{4}$$

where $\mathbf{I}_{\text{rw}}$ is the diagonal matrix with the reaction wheel inertias on its diagonal, $\boldsymbol{\omega}_{\text{rw}}$ is the column vector of the reaction wheel speeds and $\mathbf{E}_{\text{rw}}$ is the reaction wheel configuration matrix. The columns of $\mathbf{E}_{\text{rw}}$ are the unit vectors $\mathbf{e}^{\text{scf}}_{\text{i,rw}}$ along the spin axis of the reaction wheels. From (2) and (4) follows that the total angular momentum can be expressed as

$$\mathbf{h}^{\text{scf}} = \mathbf{I}_{\text{sc}}\boldsymbol{\omega}^{\text{scf}}_{\text{sc}} + \mathbf{E}_{\text{rw}}\mathbf{I}_{\text{rw}}(\boldsymbol{\omega}_{\text{rw}} + (\mathbf{E}_{\text{rw}})^t\boldsymbol{\omega}^{\text{scf}}_{\text{sc}}) \tag{5}$$

3.3.2 Angular acceleration reaction wheels

The rate of change of the absolute angular velocity of the i$^{\text{th}}$ wheel is proportional to torque acting on the wheel $\tau_{\text{rw,i}} = I_{\text{rw,i}}\frac{d}{dt}(\omega_{\text{rw,i}})_{\text{abs}} = I_{\text{rw,i}}(\dot{\omega}_{\text{rw,i}} + (\mathbf{e}^{\text{scf}}_{\text{i,rw}})^t\dot{\boldsymbol{\omega}}^{\text{scf}}_{\text{sc}})$. This can be written for all reaction wheels simultaneous in the following form

$$\dot{\boldsymbol{\omega}}_{\text{rw}} = (\mathbf{I}_{\text{rw}})^{-1}\boldsymbol{\tau}_{\text{rw}} - (\mathbf{E}_{\text{rw}})^t\dot{\boldsymbol{\omega}}^{\text{scf}}_{\text{sc}} \tag{6}$$

3.3.3 Angular acceleration of the satellite

The rate of change of the angular momentum equals the external torques acting on the satellite

$$\frac{d}{dt}(\mathbf{h}^{\text{scf}}) + \tilde{\boldsymbol{\omega}}^{\text{scf}}_{\text{sc}}\mathbf{h}^{\text{scf}} = \boldsymbol{\tau}^{\text{scf}}_{\text{ext}} \tag{7}$$

The extra cross product term $\tilde{\boldsymbol{\omega}}_{\mathrm{sc}}^{\mathrm{scf}}\mathbf{h}^{\mathrm{scf}}$ is due to the fact that the angular momentum is described with respect to the rotating SCF frame. The torque $\boldsymbol{\tau}_{\mathrm{ext}}$ consist of the sum of all external torques acting on the satellite. In particular the environmental torques and the torque due to the torque rods are included in the external torques. The external torques do specifically not include the torques generated by the reaction wheels because these are internal torques. Differentiating (5) and substitution into (7) yields

$$\mathbf{I}_{\mathrm{sc}}\dot{\boldsymbol{\omega}}_{\mathrm{sc}}^{\mathrm{scf}} + \mathbf{E}_{\mathrm{rw}}\mathbf{I}_{\mathrm{rw}}(\dot{\boldsymbol{\omega}}_{\mathrm{rw}} + (\mathbf{E}_{\mathrm{rw}})^t\dot{\boldsymbol{\omega}}_{\mathrm{sc}}^{\mathrm{scf}}) + \tilde{\boldsymbol{\omega}}_{\mathrm{sc}}^{\mathrm{scf}}\mathbf{h}^{\mathrm{scf}} = \boldsymbol{\tau}_{\mathrm{ext}}^{\mathrm{scf}} \tag{8}$$

Substitution of (6) and rearranging terms yields

$$\dot{\boldsymbol{\omega}}_{\mathrm{sc}}^{\mathrm{scf}} = (\mathbf{I}_{\mathrm{sc}})^{-1}\left(\boldsymbol{\tau}_{\mathrm{ext}}^{\mathrm{scf}} - \mathbf{E}_{\mathrm{rw}}\boldsymbol{\tau}_{\mathrm{rw}} - \tilde{\boldsymbol{\omega}}_{\mathrm{sc}}^{\mathrm{scf}}\mathbf{h}^{\mathrm{scf}}\right) \tag{9}$$

The complete dynamic model is now specified by equations (9), (6) and (5)